Autonomous Tracking Control for Four-Wheel Independent Steering Robot Based on Improved Pure Pursuit

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Abstract: Autonomous tracking control is one of the fundamental challenges in the field of robotic autonomous navigation, especially for future intelligent robots. In this paper, an improved pure pursuit control method is proposed for the path tracking control problem of a four-wheel independent steering robot. Based on the analysis of the four-wheel independent steering model, the kinematic model and the steering geometry model of the robot are established. Then the path tracking control is realized by considering the correlation between the look-ahead distance and the velocity, as well as the lateral error between the robot and the reference path. The experimental results demonstrate that the improved pure pursuit control method has the advantages of small steady-state error, fast response and strong robustness, which can effectively improve the accuracy of path tracking.

Key words: autonomous tracking control; four-wheel independent steering robot; pure pursuit; lateral error

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In recent years, with the development of artificial intelligence and market demand, the research and application of intelligent mobile robots are becoming increasingly popular[1]. Common mobile robots can be categorized into wheeled type, footed type, crawler type, etc., and different robots have their own advantages and disadvantages. Among them, the wheeled robot has the advantages such as simple structure, fast moving speed and simple control, but the ability to overcome obstacles and the ability to adapt to the environment are poor. Footed robots have the advantages of strong environmental adaptability, ability to cross obstacles, and flexible movement, but the disadvantages are low speed and low efficiency. The crawler robot has the advantages of strong terrain adaptability and strong stability. The four-wheel independent steering robot studied in this paper is a novel four wheel-legged robot. Due to its unique steering mode and wheel leg structure, it has been favored in the development and research of robots[2].

The autonomous tracking control problem of the robot is to make the robot travel on a given reference path by controlling the speed and steering of the robot. The reference path is generally given by the planning layer of the robot, and the reference path is composed of a series of path points, which need to contain information such as the position and direction of the robot. The

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autonomous tracking control of the robot is generally divided into two types: one is path tracking control\(^3\), and the other is trajectory tracking control\(^4\). Path tracking control is independent of time. In path tracking control, it can be assumed that the robot travels at a constant speed, and the driving robot approaches the reference trajectory with some control law. Trajectory tracking control is related to both time and space, requiring the robot to reach a predetermined reference path point within a specified time. Although many scholars have studied path tracking control and trajectory tracking control based on two-wheel steering, and have achieved many research results. However, there are few studies on path tracking control based on four-wheel steering\(^5\). Compared with a traditional two-wheel steering robot, the four wheels of the four-wheel independent steering robot can be independently driven and work independently, which greatly improves the maneuverability and flexibility of the robot. Therefore, relying on the developed four-wheel independent steering robot, this paper mainly studies the path tracking autonomous control of four-wheel steering.

With the development of modern control theory and intelligent control methods, robot autonomous tracking control methods show a trend of diversification. At present, the autonomous tracking control methods mainly include: PID control, fuzzy control, sliding mode control, model predictive control, and pure pursuit control. Among them, PID control is a relatively mature, stable and commonly used control method. However, because its control parameters are generally obtained by the trial and error method, it is difficult to achieve intelligent optimal control of lateral motion\(^6\). Fuzzy control fuzzy control membership function and control rule parameters need to be optimized and can not achieve the best performance of the control system\(^7\). Sliding mode control may cause oscillation or instability of the control system due to the discontinuity of its control gain\(^8\). Model predictive control relies on accurate mathematical models, while the method has a large amount of computation, and real-time problems affect its practical application\(^9\). Pure pursuit control has a long history of development for robot path tracking. It has the advantages of easy implementation, good real-time performance and mature application\(^10\). The system of our four-wheel independent steering robot is strongly coupled, time-varying and multivariate nonlinear\(^11\). Although various autonomous path tracking control methods have been well developed, their methods have certain limitations when applied to actual robot systems. In the pure pursuit, the fixed look-ahead has a great impact on path tracking, and the method does not consider the influence of the lateral error on the steering control\(^12\). In this paper, we propose a strategy to overcome at least some of these issues. The basic idea is to regard the look-ahead distance as a function of speed, and to add lateral error as input in the pure pursuit, so as to obtain more reasonable steering control. The specific implementation is first to determine the look-ahead distance according to the speed, and then calculate the steering angle by steering geometric model. At the same time, it is necessary to monitor the distance from the robot center to the reference path in real time, and further adjust the size of the steering angle according to the distance. The results show that the proposed method has good characteristics of real-time and robustness and has small path tracking error. Although pure pursuit is not a new control method, the work of dealing with pure pursuit of four-wheel steering is limited.

The remainder of this paper is organized as follows. Section 1 describes the four-wheel steering model. Section 2 introduces the pure pursuit based on the robot kinematics model. The improved pure pursuit is then described in Section 3. The experimental results and analysis are
provided in Section 4. Finally, some concluding remarks and future work directions are given in Section 5.

1 Four-Wheel Steering Model

In the current research, most steering models are vehicle steering models, that is, only two front wheels are turned, and the steering center is on the straight line of the rear axle. As shown in Fig. 1, the steering model we studied is a four-wheel steering model. The corners of the inner wheels are the same, the corners of the outer wheels are the same, and the steering center is at the center of the robot body.

From geometric relations, it can be concluded that:

\[ R = \frac{L}{2} + \frac{L}{2 \tan \delta_i} \quad (1) \]

\[ R_{out} = \sqrt{\left(\frac{L}{2}\right)^2 + \left( R + \frac{L}{2}\right)^2} \quad (2) \]

\[ R_{in} = \sqrt{\left(\frac{L}{2}\right)^2 + \left( R - \frac{L}{2}\right)^2} \quad (3) \]

\[ \delta_{in} = \tan^{-1} \frac{L}{2R + L} \quad (4) \]

where \( \delta_i \) is the turning angle of inside wheels of the robot; \( \delta_{out} \) is the turning angle of outside wheels of the robot; \( R \) is the steering radius of robotic center; \( R_{in} \) is the steering radius of inside wheels; \( R_{out} \) is the steering radius of outside wheels; \( L \) is the distance between the robot’s front wheels and the rear wheels.

The velocity relationship is presented as follows

\[ v_{out} = \frac{R_{out}v}{R} \quad (5) \]

\[ v_{in} = \frac{R_{in}v}{R} \quad (6) \]

where \( v_{in} \) is the speed of inside wheels; \( v_{out} \) is the speed of outside wheels of robot; \( v \) is the forward speed of the robot Center.

2 Pure Pursuit

2.1 Kinematic model

The robot steering kinematics model is shown in Fig. 2. \( OXY \) is the inertial coordinate frame, \( A(x_A, y_A) \) is the coordinates at the center of the robot, \( R \) is the instantaneous turning radius of the robot, \( L \) is the wheel spacing of the robot, \( \theta \) is the yaw angle of the robot, \( v \) is the speed at the center of the robot, defined as the overall forward speed of the robot, \( \delta_{in} \) and \( \delta_{out} \) are the steering angles of the inner and outer wheels of the robot respectively, defining the steering angle of the inner wheel of the robot as the steering angle \( \delta \) of the robot.

Robot kinematics is modeled as follows. At the center of the robot \( A(x_A, y_A) \), the speed along the \( X \) axis is
\[ v_x = v \cos \theta \quad (7) \]

The speed along the \( Y \) axis is
\[ v_y = v \sin \theta \quad (8) \]

The yaw rate of the robot can be obtained from the geometric relationship shown in Fig. 2:
\[ \omega = \frac{2v \tan \delta}{L(1 + \tan \delta)} \quad (9) \]

The kinematics model of the robot can be obtained from Eqs. (7)-(9) as follows
\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
\sin \theta & -\cos \theta & 0 \\
2 \tan \delta & \frac{L(1 + \tan \delta)}{\cos \delta}
\end{bmatrix}
\begin{bmatrix}
v
\end{bmatrix}
(10)
\]

2.2 Steering geometry model

Under the assumption of the above-described robot kinematics model. Fig. 3 shows the definition of the variables that define the pure pursuit control when driving forwards. \( R \) is the radius of curvature, “ref path” is the reference path, point \( A \) is the center of the robot, point \( B \) is the look-ahead point, \( L_m \) is the look-ahead distance, \( L \) is the wheel spacing of the robot, \( \delta \) is the steering angle of the robot, and \( \eta \) is the robot’s heading angle.

\[
\delta = \tan^{-1}\left( \frac{\kappa L}{2 - \kappa L} \right) = \tan^{-1}\left( \frac{L \sin \eta}{L_m - L \sin \eta} \right) \quad (12)
\]

Eq. (12) reflects the relationship between the steering angle of the robot and the heading angle of the robot, which lays a theoretical foundation for pure tracking control. The determination of the appropriate look-ahead distance becomes a key factor affecting the steering angle of the wheel.

3 Improved Pure Pursuit

3.1 Determination of look-ahead distance

The look-ahead distance is an important parameter for the robot to implement path tracking. It is especially important to adjust the look-ahead distance reasonably. A smaller look-ahead distance will make the robot track the path more accurately, while a too small look-ahead distance will cause the robot to be unstable or even oscillate. A longer look-ahead distance will make the trajectory of the robot forward more smooth, while an excessive look-ahead distance will cause the robot’s steering control to fail. Based on the above analysis, too long or too short distance is not conducive to the path tracking of the robot. It is very important to determine the appropriate look-ahead distance selection rules.

First, consider our wheeled robot experiment platform, the look-ahead distance should not be greater than the distance that the environment sensing system can detect and plan, otherwise the path tracking will fail. Second, the look-ahead distance should be a value that dynamically changes according to the actual speed of the robot. According to a large number of robot path tracking tests, the final design of the relationship between forward distance and speed is shown in Fig. 4. The formula is described as

\[
L_m(v) = \begin{cases} 
1, & v \leq 0.5 \text{ m/s} \\
2v, & 0.5 \text{ m/s} < v \leq 1 \text{ m/s} \\
2, & v > 1 \text{ m/s}
\end{cases} \quad (13)
\]

where \( L_m \) is the look-ahead distance, \( v \) is the forward speed of the robot Center.
3.2 Lateral error based control

As shown in Fig. 5, XOY is the inertial coordinate frame, o is the center of the robot, xoy is the robot coordinate system, $P_i$ is the first track point in front of the robot, and $P_i$ is the i-th track point in front of the robot, the set of path points formed by all $P_i$ is $\Omega$. $e_y$ is the lateral error of the robot, $\eta$ is the heading angle of the robot. $P_i$ is defined as the look-ahead distance point of the robot, and needs to meet the constraint $P_i \in \Omega$. After the reference path point set $\Omega$ is given in the planning layer, we should use the method of circular search to find the path point $P_i$ that is the closest to the robot and meets the condition $|oP_i| \geq L_\omega$ in the set $\Omega$. Under the condition that the formula expression of the reference path is known, the magnitude of the lateral error $e_y$ can be obtained by calculation. First, substitute the current position coordinate $x_o$ of the robot into the reference path formula to obtain the reference coordinate $y_o$ of the robot. Then the lateral error can be calculated as $e_y = y_p - y_o$.

The core of pure pursuit is to calculate the steering angle of wheel by Eq. (12), and its input variables are only heading angle and look-ahead distance. In order to improve the accuracy of path tracking, we take the lateral error into account, and let it affect the change of angle. We can now monitor the lateral error as a measurement in real time. Segmentation control is performed according to the magnitude of the lateral error. When the lateral error is in different threshold ranges, multiply the front wheel steering angle $\delta$ by a coefficient $k$ to adjust the steering angle of the robot wheel reasonably, so that the robot can realize path tracking faster and more stably.

Finally, after considering the lateral error, the heading angle and the stability and safety of the robot itself, we designed the steering controller shown as

$$
\delta = \begin{cases} 
k_1 \tan^{-1} \left( \frac{L \sin \eta}{L_{rw} - L \sin \eta} \right), & 0 < |e_y| \leq e_1 \\
k_2 \tan^{-1} \left( \frac{L \sin \eta}{L_{rw} - L \sin \eta} \right), & e_1 < |e_y| \leq e_2 \\
k_3 \tan^{-1} \left( \frac{L \sin \eta}{L_{rw} - L \sin \eta} \right), & e_2 < |e_y| \leq e_3 \\
k_4 \tan^{-1} \left( \frac{L \sin \eta}{L_{rw} - L \sin \eta} \right), & |e_y| > e_3 \end{cases}
$$

(14)

$$
\delta = \begin{cases} 
\delta, & \delta \leq 30^\circ \\
30^\circ, & \delta > 30^\circ
\end{cases}
$$

(15)

The experiments show that when $e_1 = 0.04$ m, $e_2 = 0.07$ m, $e_3 = 0.10$ m, $k_1 = 0.85$, $k_2 = 0.95$, $k_3 = 1.15$, $k_4 = 1.3$, the tracking effect is the best. This enables the vehicle to quickly reduce lateral errors and improve the accuracy of path tracking.

4 Experimental Results

4.1 Hardware system

Fig. 6 shows the four-wheel independent steering robot. The hardware system of the four-wheel independent steering robot mainly includes the energy system, the environment sensing system, the integrated navigation system, the actuator and the control system.
4.2 Straight path experiment

The straight path experiment is carried out in the campus road. The length of the straight path test is about 35 m. The experimental data acquisition process is shown in Fig. 7. Fig. 8 shows the reference path and actual path of the robot for path tracking. Fig. 9 shows the real-time variation of the lateral error and wheel steering angle of the robot under two different control methods.

Fig. 6 Four-wheel independent steering robot

Fig. 7 Data acquisition process for straight experiments

As shown in Fig. 8, the straight path tracking effects of the two methods can be visually compared. With the improved pure pursuit, the robot can eliminate large lateral errors in a short period of time and quickly enter stable linear path tracking. When tracking the straight path, we can see that the robot can adjust in time after the robot deviates from the reference path due to the disturbance of the robot or the external, so that the robot keeps track of the reference path stably.

Fig. 8 Reference path and actual path comparison

As shown in Fig. 9, the performances of the two methods on the lateral error and steering angle can be compared. From the real-time variation of the lateral error, the improved pure pursuit has a better performance than the pure pursuit. First, the improved pursuit can eliminate lateral errors in less time. Second, after the robot has eliminated the initial lateral error. Compared with the pure pursuit method, the improved pure pursuit method reduces the average lateral error from 6.43 cm to 5.46 cm, and the maximum lateral error from 14.1 cm to 11.5 cm. It can be observed from the real-time change curve of the steering angle that the improved pure pursuit performs better in the process of straight path tracking, and the amplitude of the angle...
change is smaller than that of the pure pursuit.

4.3 Curve path experiment

The curve path experiment is also carried out in the campus road. The length of the curve path test is about 26 m. The experimental data acquisition process is shown in Fig. 10. The experimental results are shown in Fig. 11 and Fig. 12. Fig. 11 shows the reference path and actual path of the robot for path tracking. Fig. 12 shows the real-time variation of the lateral error and wheel steering angle of the robot under two different control methods.

![Fig. 10 Data acquisition process for curve experiments](image)

![Fig. 11 Reference path and actual path comparison](image)

![Fig. 12 Lateral error and steering angle real-time change diagram](image)

As shown in Fig. 11, the curve path tracking effects of the two methods can be visually compared. It can be observed that in a period of time, the two methods have the same effect. But when turning, the improved pure pursuit has a smoother turning track and a smaller lateral error. When the robot has a large lateral error due to turning, the improved pure pursuit can reduce the lateral error faster, so that the robot path tracking effect is better.

As shown in Fig. 12, we can compare the performance of the two methods on the lateral error and steering angle. From the comparison curve of the real-time change of the lateral error, we can see that the lateral error of the improved pure pursuit is reduced from 8.8 cm to 8.4 cm, and the maximum lateral error is reduced from 26.3 cm to 22.8 cm. It can be seen from the real-time curve of the steering angle that the improved pure pursuit has a better performance than the pure pursuit, so the trajectory of the robot is smoother when tracking the curved path.

5 Conclusion

In this paper, a controller based on pure pursuit is designed to control the lateral motion of the four-wheel independent steering robot so that the robot can track the given path. Based on the pure pursuit controller, the lateral distance between the robot and the reference path are monitored in real time as one of the factors that influence the steering angle of the wheel. When the improved pure pursuit is utilized, the robot will have a faster dynamic response, which can quickly reduce the lateral error and reach a
steady state. Robots also have better steering characteristics for quick and smooth steering. The improved pure pursuit is suitable for path tracking research of four-wheel independent steering robots. At the same time, it has a better performance than the pure pursuit.

In future research, new methods to control the impact of lateral error on robot path tracking should be considered.

References: