Novel algorithm of gait planning of hydraulic quadruped robot to avoid foot sliding and reduce impingement

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Abstract: In order to solve kinematic redundancy problems of a hydraulic quadruped walking robot, which include leg dragging, sliding, impingement against the ground, an improved gait planning algorithm for this robot is proposed in this paper. First, the foot trajectory is designated as the improved composite cycloid foot trajectory. Second, the landing angle of each leg of the robot is controlled to satisfy friction cone to improve the stability performance of the robot. Then with the controllable landing angle of quadruped robot and a geometry method, the kinematic equation is derived in this paper. Finally, a gait planning method of quadruped robot is proposed, a dynamic co-simulation is done with ADAMS and MATLAB, and practical experiments are conducted. The validity of the proposed algorithm is confirmed through the co-simulation and experimentation. The results show that the robot can avoid sliding, reduce impingement, and trot stably in trot gait.

Key words: landing angle; gait planning; foot trajectory; friction cone; sliding; impingement

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Less than half of the Earth’s landmass is accessible to existing wheeled and tracked robots, yet people and animals using their legs can go almost anywhere[1]. Among legged robots, quadruped robot has advantages of better carrying capacity and stability compared with biped robot, and has a much simpler structure and an easier control strategy than hexapod robot or eight-legged robot; thereby quadruped robot has better comprehensive performance[2-3]. And more and more researchers are interested in studying hydraulic driving quadruped robot[4]. Up to now, the most advanced walking robot is “BigDog” made by Boston Dynamics in 2008[1]. Others like “qRT-1”[5], a quadruped robot made by Korea Institute of Industrial Technology, and “HyQ”[6], an electro-hydraulic mix-drive quadruped robot made by Italian Institute of Technology and University of Genoa, perform well.

Five domestic universities had begun the study of the hydraulic quadruped robot separately, and had achieved a lot[7]. However the total performance was not as good as “BigDog” robot. Wang[3] and Rong[4,7] studied a hydraulic quadruped robot with three degrees of freedom (DOFs) per leg, rather than the robot with four DOFs per leg which had kinematic redundancy and better adaptability to uneven terrain. In order to solve problems like foot sliding and impingement against the ground, Yoshihiro[8], Liu[9] and Wang[3] studied the composite cycloid method in foot trajectory, and reduced the impact force when foot touched the ground. Rong[7] used a cubic curve to plan the foot trajectory. But the
speed and acceleration of the foot trajectory had mutation during the support phase. Tae-Ju Kim\textsuperscript{10} proposed an energy minimization algorithm with kinematic redundancy, but the robot’s all joints of leg were actuated by hydraulic rotary actuator, not by hydraulic cylinder, and they didn’t take problems such as foot sliding and landing angle into consideration, neither. The locomotion performance of the robot was influenced by landing angle in some degree\textsuperscript{11}, and the appropriate landing angle could avoid sliding between foot end and ground.

Therefore, how to plan an effective gait is still a challenging work, which can adapt to complex terrain and avoid sliding. Then a gait planning algorithm is proposed in this paper to solve these problems. First, the foot trajectory is designated as the improved composite cycloid foot trajectory; then, the landing angle of each leg of the robot is planned to satisfy friction cone and to be controllable to improve the stability performance of the robot. Based on a geometric algorithm with controllable landing angles, the kinematics equation is derived in this paper. Finally, a dynamic co-simulation is performed with ADAMAS and MATLAB. The validity of the proposed gait planning method is confirmed through the co-simulation and practical experimentation.

1 Robot model and kinematics analysis

1.1 Robot model

The robot model, based on bionics principle, has three joints and four DOFs per leg, with a passive stretching joint, as shown in Fig. 1. The hip joint has one rolling DOF and one pitching DOF, the knee joint and ankle joint have one pitching DOF separately. All the joints are actuated by an identical linear hydraulic servo cylinder. When the robot suffers a lateral impact force, it can quickly restore balance by a lateral diagonal (trot) gait. The passive stretching joint can reduce the impact force when foot touches ground, resulting in a good cushioning effect. The torso’s material is made of steel, its total mass is 91.002 kg, for simulating the quality of pump station and other objects.

![Hydraulic quadruped robot](image1)

**Fig. 1** Hydraulic quadruped robot

Assuming the robot’s torso is parallel to the ground, and its simplified lateral view diagram is shown in Fig. 2. From Fig. 2 it can be seen that; \( \theta_1 \) is the angle between \( L_1 \) and vertical direction, i.e. the rolling angle of hip joint, it is positive when \( L_1 \) swings outside, negative when \( L_1 \) swings inside; \( \theta_2 \) is the angle between \( L_2 \) and the horizontal direction, i.e. the pitching angle of hip joint; \( \theta_3 \) is the angle between \( L_2 \) and \( L_3 \), i.e. the pitching angle of knee joint; \( \theta_4 \) is the angle between \( L_3 \) and \( L_4 \), i.e. the pitching angle of ankle joint; \( \theta_5 \) is the angle between \( L_4 \) and vertical direction, \( \theta_m \) is the angle between \( L_4 \) and the horizontal forward direction, i.e. the landing angle, \( \theta_3 \) and \( \theta_m \) have one-to-one correspondence, i.e. \( \theta_5 + 90^\circ = \theta_m \).

![Robot model](image2)

**Fig. 2** Robot model

1.2 Kinematics analysis and joints angles calculation

Since the straight line motion in a horizontal direction is the basic motion among quadruped
mammals, this paper assumes that the robot’s torso is parallel to the horizontal direction; the height of torso (H_o) keeps constant. The right foreleg (RF for short) is taken as an example to show how to calculate angles of all joints of robot. The hip joint of RF leg is taken as the origin of RF’s coordinate frame |J_1|, which is on the torso, as shown in Fig. 2. The x axis of |J_1| points to the robot’s forward direction, the z axis points opposite to the gravity direction, and y axis is determined according to right hand co-ordinates. Then the foot end coordinates to |J_1| can be obtained from the forward kinematics:

\[
J_{1xe} = L_2 \cos(\theta_2) - L_3 \cos(\theta_3 - \theta_2) + L_4 \cos(\theta_4 - \theta_3 + \theta_2)
\]

\[
J_{1ye} = \left[ -L_1 - L_2 \sin(\theta_2) - L_3 \sin(\theta_3 - \theta_2) - L_4 \sin(\theta_4 - \theta_3 + \theta_2) \right] \sin(\theta_1)
\]

\[
J_{1ze} = \left[ -L_1 - L_2 \sin(\theta_2) - L_3 \sin(\theta_3 - \theta_2) - L_4 \sin(\theta_4 - \theta_3 + \theta_2) \right] \cos(\theta_1)
\]

Then if the foot end coordinates be known, the angles of the joints can be obtained from the inverse kinematics. First

\[
\theta_1 = \tan^{-1}\left( J_{1ye} / J_{1ze} \right), \quad -90^\circ < \theta_1 < 90^\circ
\]

The relationship between foot end coordinate to |J_2| and to |J_1| is:

\[
J_{2xe} = J_{1xe}
\]

\[
J_{2ye} = ( -L_1 + J_2 x E z ) \sin(\theta_1)
\]

\[
J_{2ze} = ( -L_1 + J_2 x E z ) \cos(\theta_1)
\]

Assuming the initial coordinate of foot end of RF (the right foreleg) to |J_2| is \((x_0, 0, z_0)\), then

\[
J_{2xe} = x_0 + x_i = L_2 \cos(\theta_2) - L_3 \cos(\theta_3 - \theta_2) + L_4 \cos(\theta_4 - \theta_3 + \theta_2)
\]

\[
-J_{2ze} = -z_0 - z_i = H_0 - L_1 - z_i = L_2 \sin(\theta_2) + L_3 \sin(\theta_3 - \theta_2) + L_4 \sin(\theta_4 - \theta_3 + \theta_2)
\]

\[
\theta_4 = \theta_1 = \theta_3 + \theta_2 + \theta_5 = 90^\circ
\]

Among them, \(x_i\) in Eq. (4) and \(z_i\) in Eq. (5) are the planning foot trajectory \(x(t)\) and \(z(t)\), respectively, in the next section; \(\theta_3\) is the planning angle in the fourth section. Then, let \(A = x_0 + x_i - L_4 \sin(\theta_5), B = -z_0 - z_i - L_4 \cos(\theta_5), C = \frac{A^2 + B^2}{2L_2}\), then the solution is

\[
\theta_2 = \cos^{-1}\left( \frac{AC + \sqrt{A^2 C^2 - (A^2 + B^2)(C^2 - B^2)}}{A^2 + B^2} \right)
\]

\[
\theta_3 = \cos^{-1}\left( \frac{L_2 - A \cos(\theta_2) - B \sin(\theta_2)}{L_3} \right)
\]

\[
\theta_4 = 90^\circ - \theta_2 + \theta_3 - \theta_5
\]

The right solution can be obtained due to the mechanical constraints and constraints in normal conditions. It is similar to solve the other angles of joints of other legs.

1.3 Solution of hydraulic cylinder displacement

The hydraulic cylinder displacement can be calculated based on the cosine theorem according to joint angles and the geometric relationship between the hydraulic cylinder and the links of robot’s leg. Then the robot gait motion can be realized by controlling the hydraulic cylinder displacement through a servo valve. For example, the geometric relationship of joint \(J_4\) of RF is shown in Fig. 3, the initial length of hydraulic cylinder \(l_{41}\) is \(l_{40}\), and angles like \(\psi_{41}\), \(\psi_{42}\) and length \(l_{41}\), \(l_{42}\), which are fixed values determined by mechanical parameters of the robot. The equations of the displacement of hydraulic cylinder can be obtained as following, it is similar to solve others.

\[
\beta_4 = \theta_4 - \psi_{41} - \psi_{42}
\]

![Schematic diagram of hydraulic cylinder in geometry solver and friction cone](image)
\[ l_{44} = \sqrt{l_{41}^2 + l_{42}^2 - 2l_{41}l_{42}\cos(\beta_t)} \]  
(11)  
\[ \nabla l_{44} = l_{44} - l_{440} \]  
(12)

2 Foot trajectory planning

Quadruped animals use their legs to move back and forth in the support phase and swing phase during walking, with the foot end planning out a trajectory in space, thus completing the torso’s movement. Swing phase refers to the period of the foot end from off the ground to contacting the ground, while the support phase refers to the period of the foot end from contacting ground to off the ground\(^3\). The trajectory of foot end of robot generally needs to meet; a low contact impact and a soft landing, the track of foot end smooth, smooth continuous joint velocity and acceleration; no sliding, etc.

2.1 Trajectory planning of foot end in swing phase

There are different speeds in different positions in the cycloid equation, and it is stationary in particular places. So it is studied by many researchers\(^3,8-9\). The foot trajectory cycloid equation can be easily obtained according to the cycloid equation:

\[ x(t) = s_0 \left( \frac{2\pi}{T_y} t - \sin \left( \frac{2\pi}{T_y} t \right) \right) \]  
(13)

Its velocity and acceleration are 0 at time point \( t = 0 \) and \( t = T_y \), where \( s_0 \) is the stride length and \( T_y \) is the swing circle, which meets the requirements. Then it should be in the same form of equations in the vertical direction, but the height changes in the vertical direction: \( 0 \to h_0 \to 0 \), while stride changes in the horizontal direction: \( 0 \to s_0 \), where \( h_0 \) is the maximal raising height of foot end. Therefore, the period of the horizontal direction is twice the vertical direction, and then the equation of vertical direction is provided:

\[ z(t) = at + b\sin \left( \frac{4\pi}{T_y} t \right) + c \]  
(14)

The following constraints should be satisfied:

\[ \begin{aligned}
  z(0) &= 0 \\
  z'(0) &= 0 \\
  z''(0) &= 0 \ (0 < t \leq T_y/2) \\
  z \left( \frac{T_y}{2} \right) &= h_0 \\
  z(\frac{T_y}{2}) &= h_0 \\
  z(T_y) &= 0 \ (T_y/2 < t \leq T_y) \\
  z'(T_y) &= 0 \\
  z''(T_y) &= 0 \\
\end{aligned} \]

Solution:

\[ z(t) = \begin{cases} 
2h_0 \left( \frac{t}{T_y} - \frac{1}{4\pi} \sin \left( \frac{4\pi}{T_y} t \right) \right), & 0 \leq t < \frac{T_y}{2} \\
-2h_0 \left( \frac{t}{T_y} - \frac{1}{4\pi} \sin \left( \frac{4\pi}{T_y} t \right) \right) + 2h_0, & \frac{T_y}{2} \leq t < T_y 
\end{cases} \]  
(15)

Its velocity and acceleration in the vertical direction meet the requirements. But it’s better to let its horizontal velocity to decrease to zero before the vertical one. So \( x(t) \) can be rewritten as:

\[ x(t) = \begin{cases} 
-\frac{s_0}{2}, & 0 \leq t < \frac{1}{10} T_y \\
\frac{s_0}{2} \left( \frac{10t - T_y}{8T_y} - \frac{1}{2\pi} \sin \left( 2\pi \frac{10t - T_y}{8T_y} \right) \right), & \frac{1}{10} T_y \leq t < \frac{9}{10} T_y \\
\frac{s_0}{2}, & \frac{9}{10} T_y \leq t < T_y 
\end{cases} \]  
(16)

2.2 Single leg gait planning in gait cycle

The modified cycloid equation (13) can be brought into the motion planning in the support phase so that the speed and acceleration of \( x(t) \) in both time points \( t = T_y \) and \( t = T \) are 0 in the horizontal direction. While in the vertical direction: \( z(t) = 0 \). Let us define gait cycle \( T = 0.4 \) s, swing phase cycle \( T_y = 0.2 \) s, stride length \( s_0 = 150 \) mm, the maximal height of foot end off the ground \( h_0 = 50 \) mm.

Fig. 4a shows that \( x(t) \) changes with time in the horizontal direction. Fig. 4b shows that \( z(t) \) changes with time in the vertical direction. Fig. 4c
shows a closed foot end trajectory.

![Graph showing acceleration of z(t) vs. t/s](image)

Fig. 4 Single leg gait planning in gait cycle

Fig. 5a shows that the velocity of $x(t)$ changes with time in the horizontal direction. Fig. 5b shows that the velocity of $z(t)$ changes with time in the vertical direction. Fig. 6a shows that the acceleration of $x(t)$ changes with time in the horizontal direction. Fig. 6b shows that the acceleration of $z(t)$ changes with time in the vertical direction. All these figures show that the displacement and velocity of the closed foot end trajectory are continuous and smooth, and the robot can attain a soft landing with this closed foot end trajectory.

![Graph showing velocity of x(t) vs. t/s](image)

![Graph showing velocity of z(t) vs. t/s](image)

Fig. 5 Velocity cycle of single leg gait planning

3 Landing angle planning

It is known that a four-legged robot with four DOFs per leg has kinematic redundancy and better adaptability to uneven terrain. But its mechanical structure, motion planning and control algorithm are more complex. The planning of landing angle of robot’s foot end has two purposes: one is to simplify the motion planning and control algorithm, the other is to guide resultant forces generated by the leg into the friction cone, and the robot can better adapt to complex terrain [11-12], as shown in Fig. 3.

By assuming that the friction factor is the only consideration on robot sliding, then, the contacted force is defined as $F$, and $F_x$ represents the horizontal component of $F$, $F_z$ represents the vertical component of $F$, $u_s$ represents the coefficient of static friction. Robot will slide when $F_x$ is over the maximum static friction force $u_s F_z$, i.e. $F_x > u_s F_z$. Therefore, the appropriate landing angle can be planned according to terrain friction coefficient, terrain slope and the contacted force.

There are two ways to plan landing angle. One is to control the $\theta_2$ which is changing with time, according to Eqs. (4) (5) (6); $\theta_3, \theta_4, \theta_5$ can be obtained, thus the landing angle can be planned and the robot motion is realized. The other is planning $\theta_5$ directly, because $\theta_5$ and $\theta_m$ have one-to-one correspondence, i.e. $\theta_5 + 90^\circ = \theta_m$, as shown in Fig. 2.
The RF leg is taken as an example; the $\theta_5$ variation with time is planned. The robot moves from one terrain to another, the landing angle $\theta_m$ should be re-adjusted to improve the stability of the robot. First, $\theta_m$ should go through an adjustment period, and then $\theta_m$ will have regular changes in the gait cycle. Because $\theta_5$ and $\theta_m$ have one-to-one correspondence, here just plan the $\theta_5$ directly.

When $0 < t \leq T$, it’s $\theta_m$ adjustment cycle.

$$\theta_5 = \theta_m - (\theta_m - \theta_q) \frac{t}{T_y}, \quad 0 < t \leq T_y$$  \hspace{1cm} (17)

$$\theta_5 = \theta_q - (\theta_q - \theta_c) \frac{t - T_y}{T_y}, \quad T_y < t \leq T$$  \hspace{1cm} (18)

When $t > T$, $\theta_m$ begins normal cycle.

$$\theta_5 = \theta_c + (\theta_q - \theta_c) \frac{\text{mod}(t, T)}{T_y}, \quad 0 < \text{mod}(t, T) \leq T_y$$  \hspace{1cm} (19)

$$\theta_5 = \theta_q - (\theta_q - \theta_c) \frac{\text{mod}(t, T) - T_y}{T_y}$$

$$T_y < \text{mod}(t, T) \leq T$$  \hspace{1cm} (20)

$\theta_5$ represents the initial value of $\theta_5$, $\theta_q$ represents the required value of $\theta_5$, $\theta_c$ represents the minimal value of $\theta_5$ in the gait cycle ($T$), $T_y$ is the swing phase cycle and $T_y = 0.5 T$.

4 Simulation and experimentation

4.1 Simulation parameters setting

In this paper, the diagonal gait (trot gait) is chosen as the testing gait. The trot is a two-beat diagonal gait of the horse where the diagonal pairs of legs move forward at the same time with a moment of suspension between each beat, the motion of diagonal pairs of legs is the same in the same time$^{13,14}$. The swing phase cycle is $T_y$, and the gait cycle is $T$, the trot gait duty factor is defined as $\beta = T_y / T$. The duty factor of the trot gait used in this paper is 0.5. In order to testify the validity of the gait planning algorithm proposed in this paper, the prototype of hydraulic quadruped robot is modelled by SolidWorks software, and the simulation parameters are determined, as shown in Tab. 1, and then the co-simulation is conducted with MATLAB and ADAMS.

<table>
<thead>
<tr>
<th>Tab. 1 Simulation parameters</th>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Load</td>
</tr>
<tr>
<td>Friction coefficient</td>
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<tr>
<td>Locomotion gait</td>
</tr>
<tr>
<td>Gait cycle</td>
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<tr>
<td>Stride length</td>
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<tr>
<td>Max foot height</td>
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<tr>
<td>Simulation step size</td>
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<tr>
<td>Simulation time</td>
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</table>

4.2 Simulation

The right foreleg is taken as an example to show its all joint angle variables change, as shown in Fig. 7. $\theta_1$ keeps zero because of supposing the robot has no external swinging motion.

![Fig. 7 Joint angles of the right-fore leg](image)

From Fig. 7, it can be concluded that in the first gait cycle ($0 - 0.4$ s) the absolute values of $\theta_2$, $\theta_3$, $\theta_4$ keep increasing, it’s the robot’s standing period, as shown in Fig. 8. The second gait cycle ($0.4$ s $- 0.8$ s) is the landing angle adjustment cycle. And the other gait cycles ($t > 0.8$ s) are normal gait cycles, all the joints changing regularly, and the robot moves regularly, as shown in Fig. 9.

And then with the same simulation parameters and the same landing angle ($\theta_m = 135^\circ$), this paper chooses another foot trajectory method, a cubic curve used by Rong$^7$ for comparison;

$$x_{sw}(t) = s_0 \left( -\frac{16}{T^3} t^3 + \frac{12}{T^2} t^2 - \frac{1}{T} t - \frac{1}{4} \right)$$ and $$z_{sw}(t) = h_0 \left( -\frac{128}{T^3} t^3 + \frac{48}{T^2} t^2 \right).$$ As shown in the cubic curve
equation, the velocity of the foot end is not zero when the foot end is off or touching the ground, which would cause high impact, as shown in Fig. 10, the impact force of contact of quadruped robot with the cubic curve method is over 9 000 N and the robot always bounce when the foot end contacts the ground, while the impact force of contact of quadruped robot with the method proposed in this paper is less than 3 500 N, as shown in Fig. 11. As shown in Fig. 12 and Fig. 13, the total distance robot goes along $X$ axis is 3 090 mm (from $-2 800$ mm to $290$ mm) with the method proposed in this paper; while the total distance is 2 560 mm with the cubic curve method. These mean that the method proposed in this paper is better than the cubic curve method in low impingement and less sliding sides.

From Fig. 7 and Tab. 1, it can be known that the total distance the quadruped robot can go in 5 s is $0.4 \times 150 + (5 - 0.8)/0.4 \times 300 = 3 300$ mm. This means that there is a sliding phenomenon in trot gait with the landing angle $\theta_{in}=135^\circ$. Setting the landing angle $\theta_{in}=110^\circ$, with the same simulation parameters in Tab. 1, and the total distance the robot goes is about 3 300 mm with the method proposed in this paper, as shown in Fig. 14, which means that there is almost no sliding phe-
the friction cone condition.

![Graph](image)

**Fig. 14** Distance along x axis (with the method proposed in this paper and $\theta_{in} = 110^\circ$)

### 4.3 Experimentation

The practical hydraulic quadruped robot is in its initial testing stage, the hydraulic power system is not on the robot’s torso, and the total weight is about 80 kg including robot’s torso, legs and some steel weights. The trot gait cycle is 0.4 s, the stride length is 300 mm, and the max foot height is 50 mm. Then a comparison of the gait planning method proposed in this paper and the cubic curve method is used for testing their performance, the results are shown in Fig. 15 and Fig. 16. It is obviously that the gait planning method proposed in this paper is better than the cubic curve method in reducing impingement.

![Graph](image)

**Fig. 15** Contact force of robot (with the method proposed in this paper and $\theta_{in} = 110^\circ$)

![Image](image)

**Fig. 16** Contact force of robot (with cubic curve method and $\theta_{in} = 110^\circ$)

Fig. 17 shows the real quadruped robot developed for experimental verification. Fig. 18 illustrates the real quadruped robot in trot gait with the gait planning method proposed in this paper. When the landing angle is 135°, some sliding phenomenon will happen while the landing angle is 110°, few sliding phenomenon happen. The practical experimental results show that the real quadruped robot is stable in trot gait, and few sliding phenomenon happens with the gait planning method proposed in this paper.

![Image](image)

**Fig. 17** Real quadruped robot

![Image](image)

**Fig. 18** Real quadruped robot in trot gait

### 5 Conclusion

In order to avoid sliding and reduce impingement, the foot trajectory is designated as the improved composite cycloid foot trajectory which
can avoid sliding between foot end and ground in some degree and has lower impingement against the ground. Then the landing angle method is proposed in the following, which can avoid sliding, too. And then the kinematics equation is derived in this paper. The validity of kinematic equation and the trot gait generator is confirmed through co-simulation with ADAMS and MATLAB. Through comparisons with the cubic curve method, the validity of the gait planning method is confirmed in low impingement and avoiding sliding sides. Then the influence of the landing angle on avoiding sliding phenomenon of the quadruped robot is studied with two different landing angles. Finally the gait planning method is validated by practical experiments. Simulation and experimental results show that quadruped robot is stable in trot gait, and few sliding phenomenon happens.

References:


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